Getting Messy with Subtraction:

Making Connections Across Representations in a Thinking Classroom

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BTC BUILDING THINKING CLASSROOMS

Take care of yourself.

Take care of each other.

Take care of the space.

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When students develop flexibility with a variety of representations for mathematical ideas...

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How can we leverage BTC moves to make connections across representations?

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$$53 - 17$$

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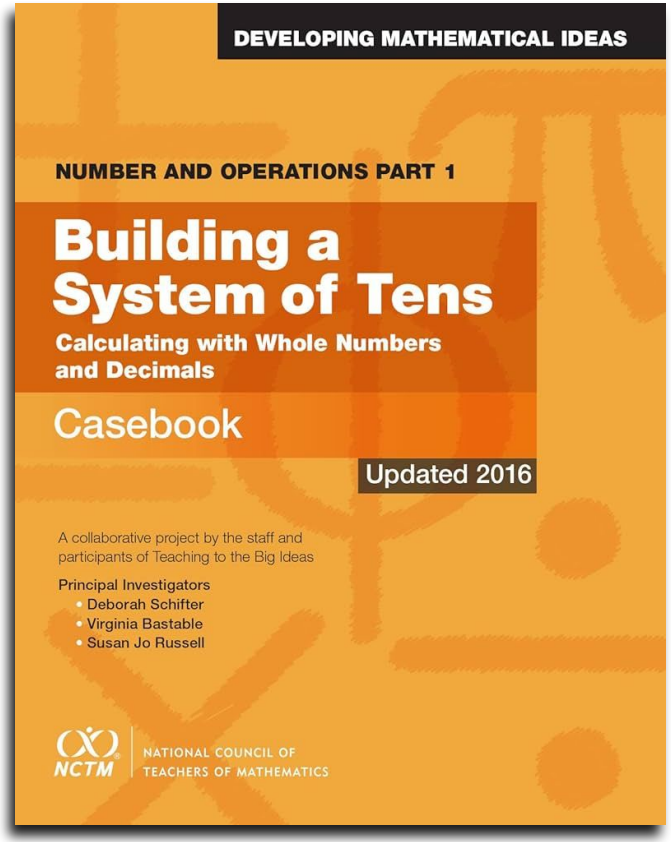
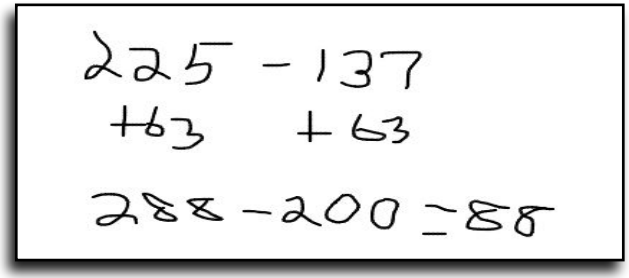
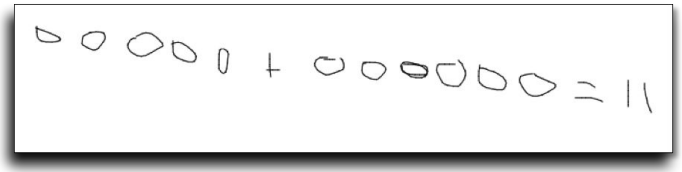


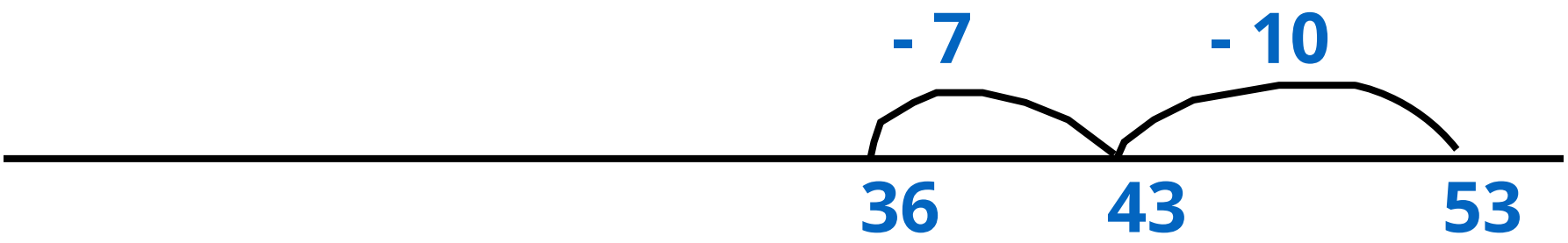


Chapter 3
Making sense of addition and subtraction algorithms



Visual Representation

53 - 17 = __
53 -10 = 43
43 - 7 = 36



Story Context

53 - 17 = __
53 -10 = 43
43 - 7 = 36

Pencils come in boxes of 10. Ezra had 53 pencils and wanted to share 17 with Olive. He gave her one full box of 10 pencils and had 43 total pencils leftover. Then he gave him 7 loose pencils.
He now has 36 pencils left.

Written Generalization

53 - 17 = __
53 -10 = 43
43 - 7 = 36

- Keep the minuend (first number) whole.
- Decompose the subtrahend (second number) by place value.
- Subtract the largest place value of the subtrahend from the minuend (first number) and record the difference.
- Subtract the next largest place value from the difference and record the new difference.
- Continue step 4 until you have subtracted all the place values.
- The final difference is the answer to the whole problem.

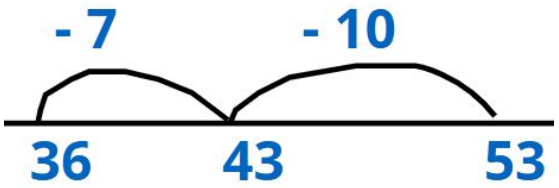
Strategy

53 - 17 = __
53 -10 = 43
43 - 7 = 36

Story context

Pencils come in boxes of 10. Sean had 53 pencils and wanted to share 17 with Collin. He gave him one full box of 10 pencils and had 43 total pencils leftover. Then he gave him 7 loose pencils.
He now has 36 pencils left.

Visual representation



Generalization

- Keep the first number whole.
- Decompose the subtrahend (second number) by place value
- Subtract the largest place value of the subtrahend from the minuend (first number) and record the difference...

SMP 2: Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

SMP 2: Reason abstractly and quantitatively

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How can we leverage BTC moves to make connections across representations?

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If we want our students to think, we need to give them something to think about.

Given one piece of the quadrant, figure out the rest.

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Strategy	 Story context Pencils come in boxes of 10. Sean had 53 pencils and wanted to share 17 with Collin. He gave him one full box of 10 pencils and had 43 total pencils leftover. Then he gave him 7 loose pencils. He now has 36 pencils left.
Visual representation	~~Generalization~~ Act it out Build it

There are two types of hints—hints that decrease challenge and hints that increase ability.

Offer hints that refer students to other quadrants.

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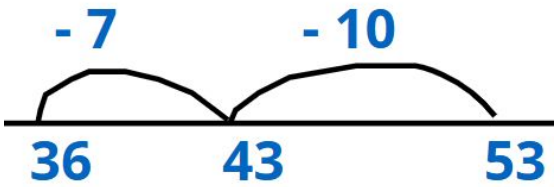
Strategy

53 - 17 = __
53 -10 = 43
43 - 7 = 36

Story context

How could your drawing help you write your story?
[WALK AWAY]

Visual representation



The goal is to use students' work, the traces of their thinking, to get the whole class to think and explain.

Consolidate by asking students to make connections across representations.

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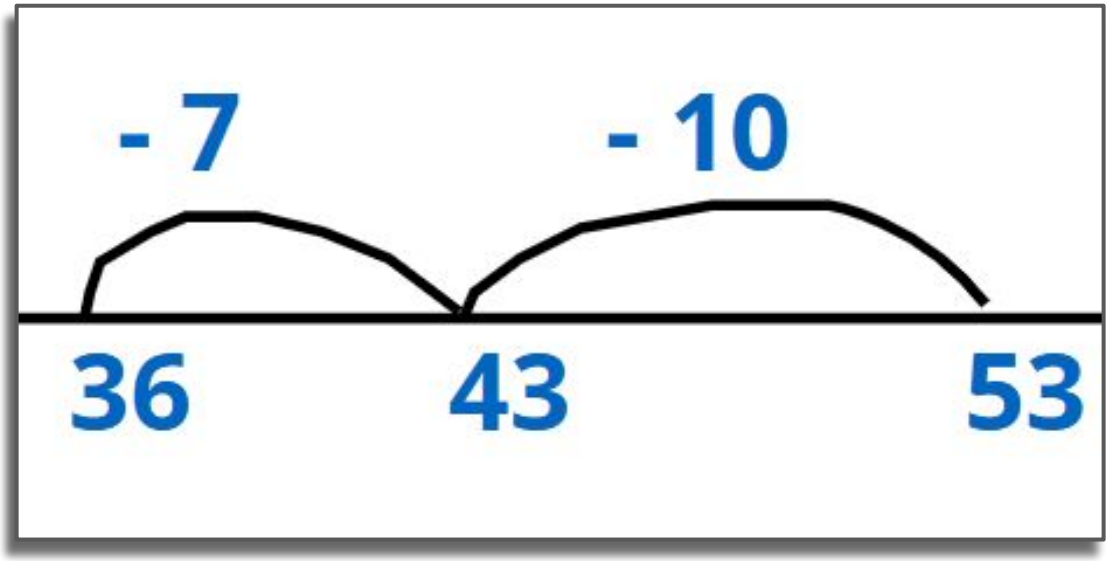
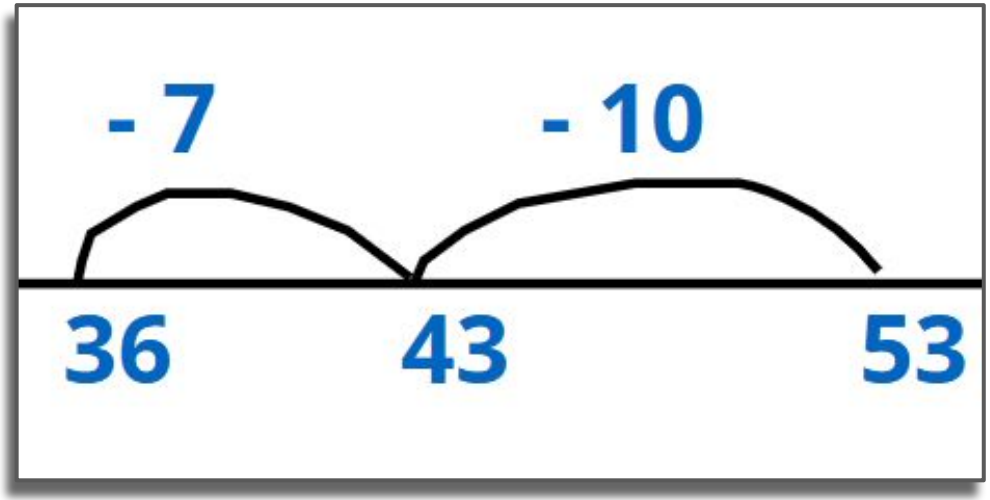


“What can we see?” or “What can we not see?”

“What is highlighted?” or “What is obscured?”



53 - 17 = __
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“Where do you see ____ in the number line?”

Given one piece of the quadrant, figure out the rest.

Offer hints that refer students to other quadrants.

Consolidate by asking students to make connections across representations.

Students can be very persistent in their efforts to get you to help them reduce their workload, and how you respond to this is important.

Rather than being the source of knowledge in the room, teachers were working to mobilize the knowledge already in the room.

You need to read the situation and know when a nod, a wink, or an encouraging remark is needed.

Purposefully mobilizing knowledge by forcing either passive or active interaction between groups activated the autonomy students had been given, increased independence, and deepened thinking.

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When students develop flexibility with a variety of representations for mathematical ideas...

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When students develop flexibility with a variety of representations for mathematical ideas, not only do they add to their own understanding, but also obtain skill in applying mathematical ideas to new areas and communicating ideas to others.

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Van de Walle, 2004

Thank you!

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