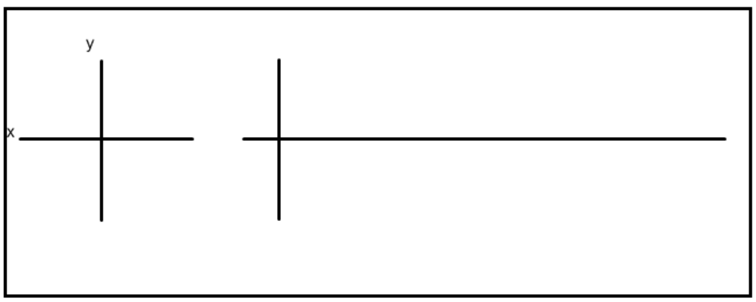
Sin Curves and Spaghetti

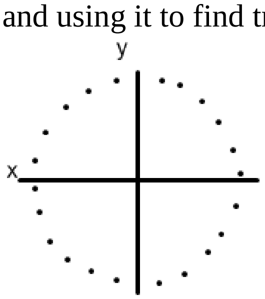
Note: I do not hand this sheet out to the students. The task is driven through conversation and careful questioning. This sheet is only for preparation; it is not intended as an instruction sheet for the students. I use the first part of this task to teach the unit circle. My students' first view of a unit circle is on this large piece of paper. A day or so later, after the students have worked problems dealing with the unit circle, we pull out these papers again and graph the actual sine and cosine curves.

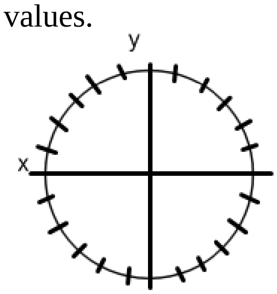
Materials: Butcher paper – 8 ft long, 1 per group
Markers, 1 per student – the more colors the better
Spaghetti, 20 pieces per group (only 7 are really needed but they are typically broken or eaten)
String – 6 ft long, 1 per group

Preparation: Ideally, the butcher paper should be cut and the two axes drawn on it before the student enter the classroom. The length of the unit circle axes should be a little more than double the length of a piece of spaghetti. The sin curve axis should take up the rest of the paper



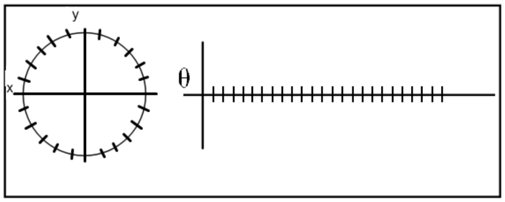
Unit Circle
The students need to create a unit circle. The unit we want to choose is the length of a piece of spaghetti. Have the students place one end of a piece of spaghetti on the origin and make a mark at the other end. Next, instruct them to rotate the spaghetti and continue making marks until they have enough to draw a circle with radius 'one spaghetti'. Next, have the students draw a circle to the best of their ability and divide that circle into 24 equal pieces using tick marks, 5 marks in each quadrant. Ask the students to label each tick mark in radians. Each mark is $\pi/12$ radians. The students are then asked to label each tick mark and then enter into a conversation about the unit circle and its uses. Each student is given a blank unit circle and asked to transfer the information from the group unit circle to their individual unit circles. The class then spends adequate time exploring the unit circle and using it to find trig values.



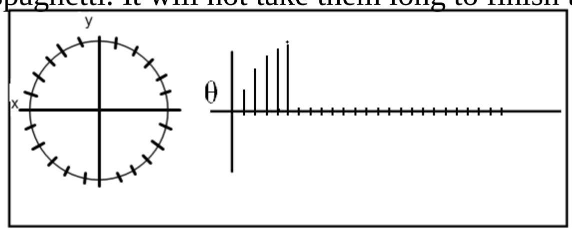


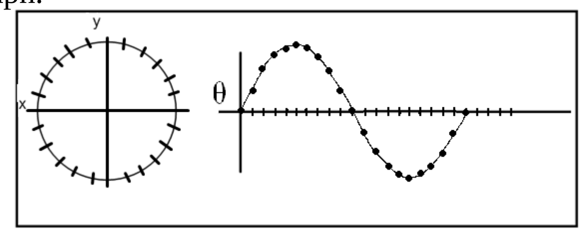
Sine Curve
At the beginning of a new day, the students are told that they will be graphing the sine function. The students already know that to graph a function they must have a domain and a range. From previous lessons they understand that the domain of a sine function is an angle and the range of a sine function is a ratio. In our discussions of the unit circle, they are also aware that when the hypotenuse is equal to 1, the ratio of the opposite side to the hypotenuse is equal to the length of the opposite side. (That's what makes the unit circle so wonderful!) Also, when we defined a radian, we wanted an angle measure that translated nicely into a length. (All of this information is invoked from the students using careful questioning. I try very hard not to tell them

much. It helps them understand both the construction and the motivators behind it.) Using the string, we are able to transfer our domain (in radians) to our new θ axis. Have the students wrap the string around the unit circles start at 0 radians and transfer the 24 tick marks on the long axis and label that axis the θ axis.



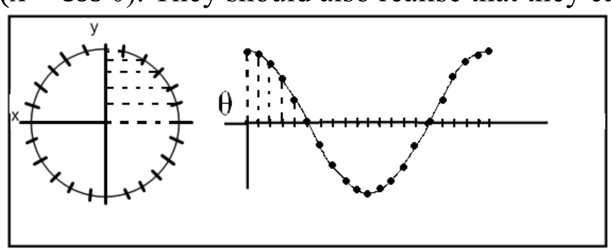
The students are then asked what the range or the 'Y' values will be. Somebody will remember that on the unit circle, sin θ = y. Visually, that corresponds the distance from the y axis to the corresponding tick mark. Thus, the distance around the unit circle is the input (domain) and the distance from the tick mark to the y axis is the output (range). This is where the spaghetti comes in. Instruct the students to break a piece to the exact length from the tick mark to the y axis. Then transfer those length onto the θ, y axes. The first spaghetti is of course length 0 and the 6th is a full length of spagetti; the other four are somewhere in betwee
The students will quickly discover that the second quadrant and the susequent two will use the same pieces of spaghetti. It will not take them long to finish the graph.





It should be obvious that the graph would continue in both directions if the students were to continue around the unit circle. Some items for discussion include: Why will the pieces of spaghetti from the first quadrant work for all of the other quadrants? What does that tell us about the sine function? Why did the graph dip below the θ axis? How long does it take for the graph to start over? Is that true for any part of the function? Is the graph continuous?

Cosine Curve
It is also very valuable to discuss the graph of the cosine curve. It will not take much questioning before the students realize that the inputs are the same but the outputs are now the distance from the tick mark to the x axis (x = cos θ). They should also realise that they can use the same pieces of spaghetti that they use for the sin θ.



Transformations: One of the most powerful lessons to be learned from this lesson is the idea of transformation. For instance, ask the students what y = sin θ + 1 really means. Some may try to revert back to the rules they attempted to memorise previously. Help them think about what that '+1' means in the context of the spaghetti. Some student will realise that it means to take the broken piece of spaghetti and add an additional full length of spaghetti; thus the graph will be simmilar to the orginal but it will be one spaghetti higher. In order to graph y = 2sin θ, each piece of spaghetti will be doubled; thus the graph will be twice as high and twice as low. y = 1/2cos θ – 2 will have the shape of the cosine curve but it will be half as tall and shifted down two full spaghetti lengths. The other trasformations such as y = sin(θ-π/4) or y = -sin θ can also be placed in context but I will leave them to be discovered by the reader.